[redacted]

COMBINATORICS

1. INTRODUCTION TO PERMUTATIONS AND COMBINATIONS

1. INTRODUCTION

$\Rightarrow n!$ (or) $\underline{n|}$

$\Rightarrow n! = 1 \cdot 2 \cdot 3 \cdots n$
$\Rightarrow 0! = 1$
} factorial of Negative numbers does not exist.

$\Rightarrow \boxed{n! = (n-1)! \times n}$

$\Rightarrow (n+1)! - n! \Rightarrow n!(n+1) - n!$

$\Rightarrow n!(n+\cancel{1}-\cancel{1})$

$\boxed{(n+1)! - n! = n*(n!)}$

$\Rightarrow (n-1)! + n! = (n-1)! + (n-1)! * n$

$= (n-1)!(1+n)$

$\boxed{(n-1)! + n! = (n+1)(n-1)!}$

$\Rightarrow {}^nP_r = \dfrac{n!}{(n-r)!}$ $\Rightarrow {}^5P_3 = \underbrace{5\times4\times3}_{(3 \text{ literals})} = 60$

$\Rightarrow {}^{10}P_4 = 10\times9\times8\times7$

$= 5040.$

$\Rightarrow {}^nC_r = \dfrac{n!}{(n-r)!\,r!}$

$\Rightarrow \boxed{{}^nP_r = {}^nC_r \times r!}$

2. PROPERITIES OF NCR-1

1) ${}^nC_r = {}^nC_{n-r}$ $\Rightarrow$ let $n-r=x$ $\Rightarrow {}^nC_r = {}^nC_x$

$$\frac{n!}{(n-r)!\,r!} = \frac{n!}{(n-x)!\,x!}$$

$$= \frac{n!}{(n-(n-r))!\,(n-r)!}$$

$$\frac{n!}{(n-r)!\,r!} = \frac{n!}{r!\,(n-r)!}$$

$\therefore \boxed{{}^nC_r = {}^nC_{n-r}}$

If ${}^nC_x = {}^nC_y$ then $\begin{cases} x=y \text{ or} \\ y=n-x \text{ or} \\ x=n-y. \end{cases}$

Now, consider ^nC_r. Now the value of ^nC_r is max iff

a) n is even $\Rightarrow \boxed{r = n/2}$

b) n is odd $\Rightarrow \boxed{r = \frac{n-1}{2} \text{ or } r = \frac{n+1}{2}}$

c) $^nC_x = {^nC_y} \Rightarrow \boxed{x+y=n}$

3. PROPERITIES OF NCR -- 2

$$\Rightarrow \boxed{^nC_r + {^nC_{r-1}} = {^{(n+1)}C_r}}$$

$$\Rightarrow \frac{n!}{(n-r)!\,r!} + \frac{n!}{(n-r+1)!\,(r-1)!}$$

$$= \frac{n!}{(n-r)!\,(r-1)! \times r} + \frac{n!}{(n-r)!\,(n-r+1)\,(r-1)!}$$

$$= \frac{n!}{(n-r)!\,(r-1)!}\left[\frac{1}{r} + \frac{1}{n-r+1}\right]$$

$$= \frac{n!}{(n-r)!\,(r-1)!}\left[\frac{n-\cancel{r}+1+\cancel{r}}{nr-r^2+r}\right]$$

$$= \frac{n!\,(n+1)}{(n-r+1)!\,r!} = \frac{(n+1)!}{(n+1-r)!\,r!} = {^{(n+1)}C_r}$$

Ex:

$$^{10}C_4 + {^{10}C_3} = {^{11}C_4}$$

$$= {^{10}C_3} + {^{10}C_4} = ?$$ we think $r=3$ but ✗ $r=4$

$$\left({^{10}C_4} + {^{10}C_3}\right) = {^{11}C_4}$$

$$^{10}C_4 + {^{10}C_3} = {^{(11)}C_{(4)}}$$

11: 1 more than 10

4: largest of 4, 3

4. PROPERITIES OF NCR-3

$$\Rightarrow {^nC_r} = \frac{n}{r}\left({^{(n-1)}C_{(r-1)}}\right)$$

$$\Rightarrow {^nC_r} = \frac{n\,(n-1)!}{(n-r)!\,(r-1)!\,r}$$

$$= n/r\left[\frac{(n-1)!}{(n-r)!\,(r-1)!}\right]$$

$$= n/r\left[\frac{(n-1)!}{((n-1)-(r-1))!\,(r-1)!}\right] = n/r\left[{^{(n-1)}C_{(r-1)}}\right]$$

$$^{10}C_3 = \frac{10!}{7!\,3!} \Rightarrow \frac{\cancel{7!} \times 8 \times 9 \times 10}{\cancel{7!} \times 3 \times 2 \times 1}$$

$$^{10}C_3 = \frac{10}{3} \times {^9C_2} = \frac{8 \times 9 \times 10}{3 \times 2 \times 1}$$

$$= \frac{10}{3} \times \frac{9}{2} \times {^8C_1}$$

$$= \frac{10}{3} \times \frac{9}{2} \times \frac{8}{1} \times {^7C_0} = \frac{10 \times 9 \times 8}{3 \times 2 \times 1}$$

5. PROPERITIES OF NCR -- 4

$$\Rightarrow \boxed{\frac{{}^{n}C_{r}}{{}^{n}C_{r-1}} = \frac{(n-r+1)}{r}}$$

$$\frac{{}^{n}C_{r}}{{}^{n}C_{r-1}} = \frac{\cancel{n!}}{\cancel{(n-r)!}\,r!} \times \frac{(n-r+1)!\,(r-1)!}{\cancel{n!}}$$

$$\boxed{\frac{{}^{n}C_{r}}{{}^{n}C_{r-1}} = \frac{(n-r+1)}{r}}$$

$${}^{n}C_{0} + {}^{n}C_{1} + {}^{n}C_{2} + \dots + {}^{n}C_{n} = 2^{n}$$

$$\left.\begin{aligned} &{}^{n}C_{0} + {}^{n}C_{2} + {}^{n}C_{4} + \dots + \dots = \\ &{}^{n}C_{1} + {}^{n}C_{3} + {}^{n}C_{5} + \dots = \end{aligned}\right\} 2^{n-1}$$

$${}^{13}C_{0} + {}^{13}C_{1} + \dots + {}^{13}C_{13} = 2^{13}$$

$${}^{13}C_{0} + {}^{13}C_{2} + {}^{13}C_{4} + \dots = 2^{12}$$

$${}^{13}C_{1} + {}^{13}C_{3} + {}^{13}C_{5} + \dots = 2^{12}$$

6. EXAMPLE ONE NCR

W.K.T = $\frac{{}^{n}C_{r}}{{}^{n}C_{r-1}} = \frac{(n-r+1)}{r}$

Given ${}^{n}C_{r-1} = 36$, ${}^{n}C_{r} = 84$, ${}^{n}C_{r+1} = 126$

what is 'n' and r?

$$= \frac{\overset{21\,7}{84}}{\underset{9\,3}{36}} = \frac{n-r+1}{r}$$

$$= 7r = 3n - 3r + 3$$

$$= \boxed{10r = 3n+3} \text{ —(1)}$$

Now, $\frac{{}^{n}C_{r+1}}{{}^{n}C_{r}} = \frac{n-(r+1)+1}{r+1}$

$$= \frac{n-r}{r+1} \;\therefore\; \boxed{\frac{{}^{n}C_{r+1}}{{}^{n}C_{r}} = \frac{n-r}{r+1}}$$

Now, $\frac{n-r}{r+1} = \frac{126}{84} = \frac{3}{2}$

$$\Rightarrow 2n - 2r = 3r + 3$$

$$\Rightarrow \boxed{2n - 3 = 5r} \text{ —(2)}$$

Solving (1) and (2) $\boxed{n=9}$ and $\boxed{r=3}$

7. EXAMPLE 2 ON NCR

$${}^{90}C_{9} + \sum_{r=0}^{10} {}^{(100-r)}C_{8} = ?$$

$$= {}^{90}C_{9} + \left({}^{100}C_{8} + {}^{99}C_{8} + {}^{98}C_{8} + {}^{97}C_{8} + {}^{96}C_{8} + {}^{95}C_{8} + {}^{94}C_{8} + {}^{93}C_{8} + {}^{92}C_{8} + {}^{91}C_{8} + {}^{90}C_{8}\right)$$

$$= {}^{n}C_{r} + {}^{n}C_{r-1} = {}^{(n+1)}C_{r} \Rightarrow {}^{90}C_{9} + {}^{90}C_{8} = \cancel{100}\;{}^{91}C_{9} = {}^{92}C_{9} = {}^{93}C_{9} - {}^{94}C_{9}$$

$$= {}^{100}C_{8} + {}^{100}C_{9} = {}^{101}C_{9}$$

8. Example 3 on NCR

$^{50}C_{20} + 3\ {}^{50}C_{21} + 3\cdot{}^{50}C_{22} + {}^{50}C_{23}$

$= {}^{50}C_{20} + {}^{50}C_{21} + {}^{50}C_{21} + {}^{50}C_{21} + {}^{50}C_{22} + {}^{50}C_{22} + {}^{50}C_{22} + {}^{50}C_{23} + {}^{50}C_{21}$

$= {}^{51}C_{21} + {}^{51}C_{22} + {}^{51}C_{22} + {}^{51}C_{22} + {}^{50}C_{23}$

$\left.\right\}$ This process doesn't work

${}^{52}C_{22} + {}^{51}C_{22} + {}^{51}C_{22} + {}^{50}C_{23} \Rightarrow {}^{51}C_{21} + {}^{51}C_{22} + {}^{51}C_{22} + {}^{50}C_{23} + {}^{51}C_{22}$

$= {}^{50}C_{20} + 3\left({}^{50}C_{21} + {}^{50}C_{22}\right) + {}^{50}C_{23}$

$= {}^{50}C_{20} + 3\left({}^{51}C_{22}\right) + {}^{50}C_{23}$

$= {}^{50}C_{20} + {}^{51}C_{22} + {}^{51}C_{22} + {}^{51}C_{22} + {}^{50}C_{23}$

This combination doesn't work

$\Rightarrow \left({}^{50}C_{20} + {}^{50}C_{21}\right) + 2\left({}^{50}C_{21}\right) + 2\left({}^{50}C_{22}\right) + \left({}^{50}C_{22} + {}^{50}C_{23}\right)$

$\Rightarrow {}^{51}C_{21} + 2\left({}^{50}C_{21}\right) + 2\left({}^{50}C_{22}\right) + {}^{51}C_{23}$

$= {}^{51}C_{21} + 2\left({}^{51}C_{22}\right) + {}^{51}C_{23}$

$= \left({}^{51}C_{21} + {}^{51}C_{22}\right) + \left({}^{51}C_{22} + {}^{51}C_{23}\right)$

$= {}^{52}C_{22} + {}^{52}C_{23} = {}^{53}C_{23}$

9. Example 4 on NCR

$1! + 4! + 7! + 10! + \dots + 301!$ what is the last digit of the sum and 2 last digits of sum?

Sol: $1! + 4! + 7! + 10! + \dots + 301!$

⇒ Any thing after 10! will contain units digit as '0'. and '00' as last 2 digits.

⇒ Now consider 5! to 7! it also contains unit digit as '0' because $5! = 120$

⇒ Now, the only thing that effects the sum is $1! + 4! = 25$.

∴ $\boxed{\text{last digit in sum} = 5}$

$1 + 24 + 5040 = 5065$ ∴ $\boxed{\text{last 2 digits} = 65}$

10. FUNDAMENTAL RULES OF COUNTING

Assume there are

3 Red balls of diff sizes
3 Black balls of diff sizes

} The no. of ways you can choose a ball out of this 6 balls is '6' ways. This problem can be subdivided into two parts as

1) The no. of ways of choosing Redball = 3w
2) The no. of ways of choosing Blackball = 3w

$\overline{6w}$

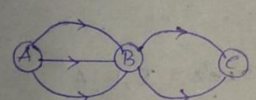
⇒ This principle / Rule is called Addition principle of probability.

2) Assume there are

(a,b)
2 Shirts
2 pants
(1,2)

} The no. of ways you can dress up is (a,1) (a,2) (b,1) (b,2) } 4 ways = (2×2)

= This principle is called multiplication theorem on probability.

⇒ 

⇒ The no. of ways you can reach from 'A' to 'C' is 3×2 = 6 ways.

⇒ 10 Blue suits
10 Black suits

} The no. of ways you can dress up = 10+10 = 20//.

11. ROW ARRANGEMENTS WITHOUT REPETETIONS

⇒ permutation = Arrangements

⇒ choosing = combination.

⇒ The no. of ways of arranging "abc" is abc, acb, bac, bca, cab, cba

⇒ $\boxed{\text{The no. of ways of arranging 'n' things in row is } n!}$

$\underset{n}{\_}\ \underset{(n-1)}{\_}\ \underset{(n-2)}{\_}\ \underset{(n-3)}{\_} \cdots \underset{1}{\_}$

$= 1 \cdot 2 \cdot 3 \cdots (n)$

$= n!$

12. EXAMPLES ON ROW ARRANGEMENTS WITHOUT REPETETIONS

1) CINEMA ⇒ The no. of letters / arrangements of this word = $6! = 720$

Restricted Permutations

⇒ The no. of words that are formed where the word always begin with 'C'.

∴ | C | 5 | 4 | 3 | 2 | 1 | ∴ $1 \times 2 \times 3 \times 4 \times 5 = 120$ words.

⇒ Start with 'C' and end with 'A'.

| C | 4 | 3 | 2 | 1 | A | $= 4 \times 3 \times 2 \times 1 = 4! = 24.$

⇒ 'C' and 'A' should be the Extremes.

| C | 4 | 3 | 2 | 1 | A | & | A | | | | | C |

$\underbrace{4\ 3\ 2\ 1}_{4!}$ $\underbrace{\quad}_{4!}$

$= 4! + 4!$

$= 48$ words.

⇒ Vowels are at even positions

1 2 3 4 5 6

3! ways

3! ways

Vowels = I, E, A

Consonants = C, N, M

$= 3! \times 3! = 36$ ways.

⇒ 'C' and I should always be together

| C I | | | | | |

5 ways   4! ways

$= 5 \times 4! \times 2!$ Internal permutations of C, I

$= 5 \times 24 \times 2$

$= 10 \times 24$

$= \underline{240}$

(or)

(CI)NEMA

Single unit $= 5! \times 2!$

$= 240$

⇒ C, I, N should always be together

⇒ $\boxed{C \mid I \mid N} \mid \; \mid \; \mid \;$ (1 group)

$4! \times 3! = 24 \times 6 = 124$

4 groups = 4! ways.

↳ Internal permutations

⇒ 'C' and 'I' should never be together = Total Arrangements − CI come together

⇒ $\boxed{6! - (5! \times 2!)} = 480$

⇓

This procedure is applicable only for 2 letters not for 3 Alphabets.

192 ⇐ $4 \times 2! \times 4!$ ⇐ | C | | I | | | |

144 ⇐ $3 \times 2! \times 4!$ ⇐ | C | | | I | | |

96 ⇐ $2 \times 2! \times 4!$ ⇐ | C | | | | I | |

48 ⇐ $1 \times 2! \times 4!$ ⇐ | C | | | | | I |

480

⇒ C I N should always be together and 'I' should always be 'C' and 'N'.

| C | I | N | | | |

$2 \times 4 \times 1 \times 3! = 4 \times 6 = 24$ words $\times$ 2 → (C, N can be changed).

= 48 words

13. ROW ARRANGEMENTS WITH REPETETIONS

Arrangements with Repetetion. (some objects are identical)

The no. of ways of arranging 'n' elements in a row in which 'r' elements are identical is ~~called~~ $= \frac{n!}{r!}$

⇒ we have 'n' objects out of which 'm' are of one kind, 'r' are of another kind and 'p' objects are of other kind, then the no. of ways to arrange them is " $\frac{n!}{m!\, r!\, p!}$ "

14. EXAMPLES ON ROW ARRANGEMENS WITH REPETETIONS - 1

RAVINDRABABU

R - 2 times

A - 3 times

B - 2 times.

⇒ NO of 12 lettered words that canbe formed is

$$\frac{12!}{2!\,3!\,2!}$$

Restricted Permutations ⇒ ⇒ start with R

R											

$$\frac{11!}{3!\,2!}$$

⇒ start with 'R' and end with 'U'

R											U

$$\frac{10!}{3!\,2!}$$

⇒ Vowels and consonents should come together.

Vowels = A, I, A, A, U

consonants = R, V, N, D, R, B, B.

} 2 Groups 2! ways

$$2! \times \frac{5!}{3!} \times \frac{7!}{2!\,2!}$$

15. EXAMPLES ON ROW ARRANGEMENTS WITH REPETETIONS -- 2

RAVINDRABABU.

R - 2 times

B - 2 times

A - 3 times.

⇒ All a's should come together

$\boxed{AAA}$ - - - - - - - - - -

$$= \frac{10!}{2!\,2!} \times \frac{3!}{3!} = \frac{10!}{2!\,2!} = \begin{pmatrix} AAA\,BB \\ \text{or} \\ AAA,\,B-B \end{pmatrix}$$

⇒ All A's should be together and B's should not be together

$= \boxed{AAA}$

= (All A's coming together) - (All B's not coming together).

$$= \left(\frac{10!}{2!\,2!} - \frac{9!}{8!} \times \frac{3!}{3!} \times \frac{2!}{8!}\right) \quad \boxed{AAA}\boxed{BB}\ R, V, I, N, D, R, U$$

16. EXAMPLES ON

R's → not together

B's → should not be

Represen

together &

on 'B'

∴ $\frac{12!}{2!\,3!\,2}$

∴ The N

17. EXAMPLE

2, 3, 4, 4, 5

18. EXAMPL

2, 3, 4, 4, 5.

⇒ Even dig

16. EXAMPLES ON ROW ARRANGEMENTS WITH REPETETIONS - 3

R's → not together

B's → should not be together

[Total methods] − [(No. of Arrangements in which R, B come together) + (R's not together & B's come together) + (R together & B - not together)]

(RR, BB)
(R–R, BB)
(RR, B–B)
(R–R, B–B)

Represent R's come together & no Restriction on 'B'.

$$\therefore \frac{12!}{2!\,3!\,2!} - \left[\left[\frac{11!\times 2!}{2!\,3!\,2!}\right] + \left(\frac{11!}{3!\,2!} - \frac{10!}{3!}\right)\right]$$

→ Total − (R come together

$$= \left(\frac{11!}{3!\times 2!} - \frac{10!}{3!}\right)$$

$\therefore$ The No. of ways $= \dfrac{12!}{2!\,3!\,2!} - \left[\dfrac{11!\times 2!}{2!\,3!\,2!} + \left(\dfrac{11!}{3!\,2!} - \dfrac{10!}{3!}\right)\right]$

17. EXAMPLES ON ROW ARRANGEMENTS WITH REPETETIONS - 4

2, 3, 4, 4, 5

⇒ Total no. of No's $= \dfrac{5!}{2!} = 60$

⇒ Even no's $=$ _ _ _ _ 2 (2, 4, 4) $\dfrac{4!}{2!} = 12$ | _ _ _ _ (2, 4) $\dfrac{4!}{1!} = 24$

⇒ 36 even no's.

= odd nos: _ _ _ _ 3 $\dfrac{4!}{2!}$ | _ _ _ _ 5 $\dfrac{4!}{2!}$

$= \dfrac{2\times 4!}{2!} = 24$ odd no's.

18. EXAMPLES ON ROW ARRANGEMENTS WITH REPETETIONS - 5

2, 3, 4, 4, 5.

⇒ Even digits should be at odd places. (2, 4, 4)

2!

_ _ _ _ _

2 4 4 → $\dfrac{3!}{2!}$

$\therefore \dfrac{3!}{2!} \times 2! = 6$ ways.

⇒ Find the no. of no's that are < 40,000

$\underset{2}{\underset{2,3}{\boxed{}}}\ \underbrace{\boxed{}\boxed{}\boxed{}\boxed{}}_{\frac{4!}{2!}}$ $= \frac{2 \times 4!}{2!} = 24$ words.

⇒ The no's should be < 50,000

$\boxed{}\boxed{}\boxed{}\boxed{}\boxed{}$

2/ — Fix it then → Now, $\frac{4!}{2!}$ no's

3/ — Fix it then → Now, $4!/2!$ no's

4 — Fix it then → Now, $4!$ no's.

$= 2\left(4!/2!\right) + 4!$

$= 48$

19. EXAMPLES ON ROW ARRANGEMENTS WITH REPETETIONS – 6

2, 3, 4, 4, 5

→ Find the 5 digited no's which are ÷ble by 2. = 36 even no's.

⇒ ÷le by 3 ⇒ (A no' is ÷ble by 3 if the total sum is ÷ble by 3)

$\boxed{}\boxed{}\boxed{}\boxed{}\boxed{}$ $2+3+4+4+5 = 18$

∴ The no's are $= \frac{5!}{2!}$

⇒ Divisible by 4 (last 2 digits should be ÷le by 4)

= _ _ _ _ _

_ _ _ (2,4) = $3!$

_ _ _ (4,4) = $3!$

_ _ _ (3,2) = $3!/2!$

_ _ _ (5 2) = $3!/2!$

∴ The total no's = $3! + 3! + 2(3!/2!)$

$= 3(3!)$

$= 18$

⇒ ÷le by 5

$\underbrace{\_\ \_\ \_\ \_}_{\frac{4!}{2!}}\ \underline{5}$

$\frac{4!}{2!} \times 1$ ∴ The no. of words = 12

20. EXAMPLE

0, 2, 3, 4, 5 ⇒

⇒ How man

4

(2,3,4)

3

20. EXAMPLES ON ROW ARRANGEMENTS WITH REPETETIONS

0, 2, 3, 4, 5 ⇒ Find no. of 5 digited Number

⇒ $\square\square\square\square\square$ = (Total No's) − (No's having '0' in first place)

= $5! - 4!$

= $\underline{\underline{96}}$.

(or)

$\underbrace{\square}_{\substack{(2,3,4,5)\\4}}\underbrace{\square\square\square\square}_{4!}$ = $4 \times 4!$ no's

= $\underline{\underline{96 \text{ no's}}}$.

⇒ Find the total no. of 5 digited no's that are even?

= $\square\square\square\square\underbrace{\square}_{(0,2,4)=3\text{ chances}}$.

Now, $\underbrace{\square\square\square\square}_{\text{No's}=4!\text{ no's}}\boxed{0}$ $\quad$ $\underbrace{\square}_{\substack{(3,4,5)\\3}}\underbrace{\square\square\square}_{3!}\boxed{2}$ ∴ $\boxed{\text{no's} = 3\times 3!}$

$\underbrace{\square}_{\substack{(2,3,5)\\3}}\underbrace{\square\square\square}_{3!}\boxed{4}$ = $3 \times 3!$ No's

∴ Total No's = $4! + (3\times3!) + (3\times3!)$

= $24 + 3(6) + 3(6)$

= $24 + 36$

= $\underline{\underline{60}}$.

⇒ How many 5 digited no's are present that are divisible by 5?

$\underbrace{\square\square\square\square}_{4!}\boxed{0}$ = $4!$ No's

⇒ $\boxed{\text{Total No's} = 4! + 3 * 3!}$

$\underbrace{\square}_{\substack{(2,3,4)\\3}}\underbrace{\square\square\square}_{3!\text{ ways}}\boxed{5}$ = $3\times3!$ No's

⇒ Find no. of 5 digited no's that are divisible by 4.

$\underbrace{\square\square\square}_{3!\text{ ways}}\underbrace{\square\square}_{6\text{ ways}}$

Given no's = 0,2,3,4,5

(20) (40)
(24) (52)
(32)
(04)

$\underbrace{\square\square\square}_{3!}\underbrace{\boxed{2}\boxed{0}}_{1} = 3!$

$\underbrace{\square\square\square}_{3!}\underbrace{\boxed{0}\boxed{4}}_{1} = 3!$ no's

$\square\underbrace{\square\square}_{2!}\boxed{2}\boxed{4} = 2\times2!$ (2)

$\underbrace{\square\square\square}_{3!}\boxed{4}\boxed{0} = 3!$

$\square\underbrace{\square\square}_{2!}\boxed{3}\boxed{2} = 2\times2!$ (2)

$\square\underbrace{\square\square}_{2!}\boxed{5}\boxed{2} = 2\times2!$ (2)

∴ Total no's divisible by 4 = $3(3!) + 3(2\times2!)$

$= 3(6)+3(4)$

$= 18+12 = \underline{30}$

21. RANK IN A DICTIONARY

⇒ RAVI — 4! words can be formed. (Find the rank of RAVI)

⇒ A I R V — Alphabetical order

⇒ A _ _ _ = 3! words

I _ _ _ = 3! words

$\boxed{R}\boxed{A}$ I V = 1! word

R A V I = 1 word

14

⇒ RANK (Find the rank of the word Rank)

AKNR

A _ _ _ = 3!

K _ _ _ = 3!

N _ _ _ = 3!

~~R A~~

$\boxed{R}\boxed{A}$ K _ — 1! word

$\boxed{R}\boxed{A}\boxed{N}\boxed{K}$ — 1 word

20

∴ Rank of the word RANK = $\underline{20}$.

(12)

⇒ Find the Rank of sourav. = 6! words are formed

⇒ Alphabetical order = A O R S U V

(13)

A – 5!

O – 5!

R – 5!

$\boxed{S}$ A = 4!

$\boxed{S}\boxed{O}$ A = 3!

$\boxed{S}\boxed{O}$ R = 3!

$\boxed{S}\boxed{O}\boxed{U}$ A = 2!

$\boxed{S}\boxed{O}\boxed{U}\boxed{R}\boxed{A}\boxed{V}$ = 1

$\boxed{399}$

⇒ Find the Rank of IGNITE

Alphabetical order = E G I I N T

E – – – – – = $5!/2!$ = 60

G – – – – – = $5!/2!$ = 60

$\boxed{I}$ E – – – – = 4! = 24

$\boxed{I}\boxed{G}$ E – – – = 3! = 6

$\boxed{I}\boxed{G}$ I – – – = 3! = 6

$\boxed{I}\boxed{G}\boxed{N}$ E – – = 2! = 2

$\boxed{I}\boxed{G}\boxed{N}\boxed{I}$ E.T – 2! = 2

$\boxed{I}\boxed{G}\boxed{N}\boxed{I}\boxed{T}\boxed{E}$ = 1

160

22. RANK IN A DICTIONARY-2

⇒ Given the Alphabets A, I, R, V Find the word whose rank is 16

Now find the word whose rank is 14?

A! A _ _ _ = 3! = 6 (1-6)

I _ _ _ = 3! = (7-12)

R A _ _ = 2! (13/14)

RAIV — 13

$\boxed{\text{RAVI} = 14\text{th}}$

⇒ Find the word whose rank is 20.

A _ _ _ = 3! = 6 (1-6)

I _ _ _ 3! = 6 (7-12)

R _ _ _ 3! = 6 (13-18)

$\boxed{V}$ $\boxed{A}$ _ _ = 2! (19,20)

VAIR − 19

$\boxed{\text{VARI} = 20}$

Rank = 18

A _ _ _ = 3! = 6

I _ _ _ = 3! = 6

$\boxed{R}$ $\boxed{A}$ _ _ = 2! = 2

$\boxed{R}$ $\boxed{I}$ _ _ = 2

$\boxed{R}$ $\boxed{V}$ _ _ = 2

R V A I = 17

R V IA = 18 ∴ $\boxed{\text{RVIA} = 18}$

⇒ Find the word whose rank is 160. (E, G, I, I, N, T)

E _ _ _ _ _ = 5!/2! = 60 (1-60)

G _ _ _ _ _ = 5!/2! = 60 (61-120)

$\boxed{I}$ E _ _ _ _ = 4! = 24 (121-144)

× $\boxed{I}$ $\boxed{G}$ _ _ _ _ = 4! = 24 (145 − 168) '160' is in the range so our Reqd word starts with IG.

$\boxed{I}$ $\boxed{G}$ E _ _ _ = 3! = 6. (145-150)

$\boxed{I}$ $\boxed{G}$ I _ _ _ = 3! = 6 (151-156)

$\boxed{I}$ $\boxed{G}$ N _ _ _ = 3! = 6 (excees 160). ∴ The word starts with IGN

$\boxed{I}$ $\boxed{G}$ $\boxed{N}$ E _ _ = 2! = (157-158)

$\boxed{I}$ $\boxed{G}$ $\boxed{N}$ $\boxed{I}$ $\boxed{T}$ $\boxed{E}$ = 160

23) CIRCULAR

⇒ The no. of ar

a,b,c

abc
acb
bac
bca
cab
cba.

24. CIRCULA

How many

a circle?

RAVINDRA

CIRCULAR :

25 EXAMPL

A Family

how many w

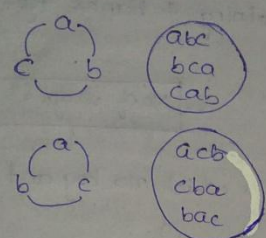
and father

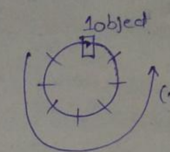
23) CIRCULAR PERMUTATIONS WITHOUT REPETETIONS.

⇒ The no. of arrangements of 'n' distinct objects around a circle is $(n-1)!$

a,b,c

abc
acb
bac
bca
cab
cba.





$(n-1)!$ ways.

∴ c/r permutations of 'n' distinct objects = $(n-1)!$

24. CIRCULAR PERMUTATIONS WITH REPETETIONS

How many ways you can arrange letters of the word RAVINDRA? In a circle?

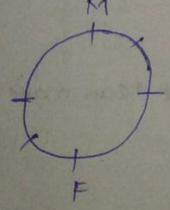
RAVINDRA = 8 letters ⇒ No. of Arrangements = $7!$ / A, R are repeated two times

$$\therefore \frac{7!}{2!\,2!}$$

CIRCULAR : $\frac{8!}{2!\,2!}$ PERMUTATIONS : $\frac{12!}{2!}$

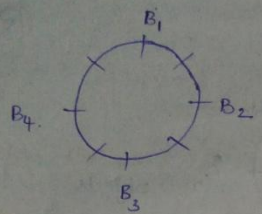
25. EXAMPLES ON CIRCULAR PERMUTATIONS -- I

A family has 6 mem, Mother, Father, Sons $(S_1 S_2)$, Daughters $(D_1 D_2)$, Now how many ways you can make them sit in c/r table such that Mother and Father faces each other.



Then 4 mem are remaining and 4 objects are present ∴ $4!$ ways.

3) We have 4 Boys – $B_1 B_2 B_3 B_4$, 4 Girls – $G_1 G_2 G_3 G_4$ } what are the no. of ways in which no two boys are together and no two girls are together.



⇒ 4 boys can be arranged in circle in $3!$ ways

⇒ In the remaining 4 blanks 4 girls can be seated in $4!$ ways

∴ $\boxed{\text{No of ways} = 4! \times 3!}$

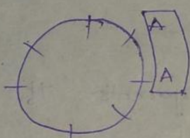
⇒ No. of ways in which no boy and Girl are together is $7! - (4! \times 3!)$

26. EXAMPLE ON CIRCULAR PERMUTATIONS -- 2

RAVINDRA ⇒ Arrange in circle and ① Two a's always come together

⇓

$\frac{8!}{2!\,2!}$ ways.



$= \frac{6!}{2!}$

$\underbrace{\text{R V I N D R } \boxed{AA}}_{\text{7 objects}}$

6! ways

'R' is repeated two times.

⇒ Two a's should never come together.

$$\boxed{\text{No 2 a's come together} = \left(\frac{8!}{2!\,2!}\right) - \left(\frac{6!}{2!}\right)}$$

27. EXAMPLES ON GARLAND MODEL QUESTIONS

The no. of ways in which we can make garland from 'n' flowers is $\frac{(n-1)!}{2}$.

There are 5 flowers Now the no. of ways that I can make garland is $\frac{(n-1)!}{2} = \frac{(5-1)!}{2} = \frac{4!}{2} = \underline{\underline{12}}$

28. INTRODUCTION TO COMBINATIONS

The no. of ways of choosing 'r' objects from a set of 'n' distinct objects is '^nC_r' $= \frac{n!}{(n-r)!\,r!}$

$^3C_3 = 3 =$ abc
- ab
- bc
- ac

} 3 ways.

$^5C_2 = 10 =$ abcde
- ab, ac, ad, ae
- bc, bd, be
- cd ce, de

29. EXAMPLE ON COMBINATION -- 1

10 Boys, 8 Girls. In how many ways you can choose 5 of them such that the group should contains 3 Boys and 2 Girls?

$= {}^{10}C_3 \times {}^8C_2$

= Majority are Boys $\Rightarrow$ (3 Boys + 2G) + (4B + 1G) + (5B, 0G)

$= ({}^{10}C_3 + {}^8C_2) + ({}^{10}C_4 + {}^8C_1) + ({}^{10}C_5)$

$\Rightarrow$ Majority are Girls = (3G + 2B) + (4G + 1B) + (5G + 0B)

$= ({}^8C_3 + {}^{10}C_2) + ({}^8C_4 + {}^{10}C_1) + ({}^8C_5)$

$\Rightarrow$ We have 22 players, form a team of 11, we have 12 Batsman, 8 Bowlers, 2 wicket Keepers, the team should contain (5 Batsmen, 5 bowlers, 1 WK) atleast.

12	8	2WK	
BATSMAN	BOWLERS		ways
$^{12}C_5$	$^8C_5$	$^2C_1$	$^{12}C_5 \times {}^8C_5 \times {}^2C_1$
$^{12}C_6$	$^8C_4$	$^2C_1$	$^{12}C_6 \times {}^8C_4 \times {}^2C_1$
$^{12}C_6$	$^8C_5$	1	$^{12}C_6 \times {}^8C_5 \times 1$
⋮	⋮	⋮	⋮

30. EXAMPLE ON COMBINATIONS -- 2

⇒ A mathematics Question paper contains 12 Questions and we are supposed to ans 8 Then how many ways you can answer the Question paper? 12_{c_8}

⇒ The QP has 2 parts, the first 2 parts contains Questions from (1-6) and 2nd part contains Questions from (7-12). The Restriction is you have to answer atleast 3 from part 1 and In total you have to ans 8 Questions?

The no. of ways are,

(1 2 3 4 5 6) (7 8 9 10 11 12)
PART 1 PART 2.

⇒ $6_{c_3} \times 6_{c_5}$ =

⇒ $6_{c_4} \times 6_{c_4}$ =

⇒ $6_{c_5} \times 6_{c_3}$ =

⇒ $6_{c_6} \times 6_{c_2}$ =

The total ways are
$(6_{c_3} \times 6_{c_5}) + (6_{c_4} \times 6_{c_4}) + (6_{c_5} \times 6_{c_3}) + (6_{c_6} \times 6_{c_2})$

31. EXAMPLES ON COMBINATIONS -- 3

There are 10 couples (10m, 10w) Now, i want to form a team of 4 mem such that i have (2m and 2w) then how many ways we can form it.

No. of ways $\boxed{10_{c_2} \times 10_{c_2}}$

⇒ Now In the team of 4 people I should see that no couple is included.

⇒ $\boxed{10_{c_2} \times 8_{c_2}}$

⇒ Now, In the 4 mem team we need to include exactly one couple then

⇒ Now choose a couple from the 10 Couples = 10_{c_1} (I have included 2 mem since i have chosen a couple)

⇒ Now, among the $\boxed{9m \text{ and } 9w}$

Choose 1 men = 9_{c_1}

⇒ Since I need to Include exactly one couple the I should not include the wife of the person choosen above, so im left with 8 mem, so choose 1 from them = 8_{c_1} ways.

$\therefore$ $\boxed{\text{The total no. of ways} = 10_{C_1} \times 9_{C_1} \times 8_{C_1}}$

$\Rightarrow$ Now In the 4 member team, If H_1 is selected then W_5 should be selected.

case 1: H_1 is selected $\Rightarrow W_5$ should be included, Now, 9m, 9w are left $\Rightarrow$ No. of ways of selecting = $\boxed{9_{C_1} \times 9_{C_1}}$

case 2: H_1 is not included then, among 9 men choose 2 = 9_{C_2} ways.

$\Rightarrow$ If H_1 is not selected it doesnt mean that you should not select W_5, W_5 may or maynot be selected. $\Rightarrow 10_{C_2}$

$$\boxed{\text{ways} = 9_{C_2} \times 10_{C_2}}$$

$\therefore$ No. of ways = case 1 + case 2

$$\boxed{\text{ways} = 9_{C_1} \times 9_{C_1} + 9_{C_2} \times 10_{C_2}}$$

$\Rightarrow W_2, W_3$ should not be selected simultaneously.

W_2	W_3	
✓	✓	$= 10_{C_2}$
✓	×	$= 10_{C_2} \times 8_{C_1}$
×	✓	$= 10_{C_2} \times 8_{C_1}$
×	×	$= 10_{C_2} \times 8_{C_2}$

These are the cases that we need

$$= 10_{C_2}(8_{C_1} + 8_{C_1} + 8_{C_2})$$

32. EXAMPLES ON COMBINATIONS -- 4

$\Rightarrow$ There are 10 couples (10m, 10W) How many different ways we can have shakehands between them?

$\Rightarrow$ There are 20 people and we can have shakehands between any two persons = $\therefore$ The total no. of shake hands = 20_{C_2}

$$= \frac{20 \times 19}{2} = 190 \text{ Shake Hands.}$$

⇒ Find the no. of shake hands that doesnot include a couple

No. of shakehands = $\left({}^{20}C_2 - 10\right)$ → No. of shakehands of couples
10 - couples = 10 - shakehands

⇒ Let us say $H_1\ H_2\ H_3 - - - H_{10}$, $w_1\ w_2\ w_3 - - - w_{10}$ } The condition is that there should not be hand shake between prime nos

$H_2\ H_3\ H_5\ H_7$
$w_2\ w_3\ w_5\ w_7$

There shouldnot be shakehands between these problems.

∴ The No. of shakehands = $\left({}^{20}C_2 - {}^{8}C_2\right)$

33. REPETETIONS IN COMBINATIONS

$aab = 3!/2! = 3$ nos.

$- - - ({}^{3}C_2 \times 1)$

$= (3$ ways$)$

$aabcd = 5!/2!$

$= {}^{5}C_2 \times 3!$ (take two objects and fix them aa and remaining objects can be placed in 3! ways.

There are 'n' numbers/objects in which 'r' are of similar kind then the no. of words that can be formed $= \dfrac{n!}{r!} = ({}^{n}C_r)(n-r)!$ → Arrange the Remaining ones

${}^{n}C_r$ → choose the Repeating objects

34. ORDER

⇒ we have abc I have to form all 3lettered words suchthat a should occur before B. ab, a_b. → 1 letter between a and b. ab → '0' letter between ab.

a	b	c

3 positions

c a b –
a c b –
a b c –

} 3 ways.

⇒ 1st choose 2 positions from available 3 positions = ${}^{3}C_2$ (12 – abc, 13 – acb, 23 – cab) } 3

⇒ PERMUTATIONS (The condition is that vowels should always be in Alphabetical order

vowels = (AEIOU) ← This is the order in which vowels should occur

⇒ Now choose 5 positions among 11 positions = $^{11}C_5$ ways (vowels cannot be permutated because order is specified here) and place the vowels.

⇒ Remaining '6' letters can be placed in $\frac{6!}{2!}$ ways ('T' Repeated 2 times)

$\therefore$ No of ways = $\frac{11!}{\cancel{6!}\,5!} \times \frac{\cancel{6!}}{2!}$

$$\boxed{\text{No of ways} = \frac{11!}{5!\,2!}}$$

⇒ We have 'n' different letters and we are supposed (we want) to find all 'n' lettered words in which 'r' letters should be in some order then no of words = $n!/r!$

35. EXAMPLE ON ORDER

RAVIND ⇒ The no of possible words that can be formed in which 'R' should always occur before A?

⇒ Fix (R, A) ⇒ out of 6 positions choose any two = $^6C_2 \times 4!$ Remaining words.

$= \frac{6!}{2!\,4!} \times 4! = 6!/2!$

⇒ 'R' should come before 'A' and 'A' should come before V.

$R \to A \to V \Rightarrow {^6C_3} \times 3!$ → Remaing words arrangements.

⇒ R should come before A and V ⟶ $(R \to A \to V) - 6!/3!$ or $(R \to V \to A) - \frac{6!}{3!}$

~~R→A → 6!/2!~~

~~R→A~~

R . . . . . → 4!

4 ways . R . . . = $^4C_2 \times 2!$

$\therefore \ 6!/3! + 6!/3! =$ ways.

36 EXAMPLES ON ORDER-2

RAVIND ⇒ Find All words such that 'R' occurs before A or 'R' occurs before 'B'?

⇒ Find the total words and substract the ways in which 'R' and ... occur before R. ($A \to V \to R$, $V \to A \to R$)

$$= 6! - \left(\left({}^6C_3 \times 3!\right) + \left({}^6C_3 \times 3!\right)\right)$$

37. SELECTIONS WITH REPETETIONS

$abcd = {}^4C_2 = 6$ $\quad$ $aabc = {}^4C_2$

= ab
ac
ad
bc
bd
cd

aa
ab
ac
bc

$\Big\} \; 4 = {}^4C_2$

38. EXAMPLE ON SELECTIONS WITH REPETETIONS

⇒ aabc ⇒ Find/choose two objects

case 1: Two letters are same – oneway = ${}^2C_2$

case 2: Two letters are different = ${}^3C_2$ = 3 ways = (a, b, c)

Total = 4 ways

⇒ RAVINDR ⇒ Choose two letters from this name

R – 2
A – 1
V – 1
I – 1
N – 1
D – 1

case 1: Both the letters are same = 1 way.

case 2: Both the letters are different = ${}^6C_2$.

Total = ${}^6C_2 + 1$ way

⇒ choose 3 letters from the given letters.

Case 1: 3 different letters = ${}^6C_3$.

Case 2: '2' same + 1 different = $1 * {}^5C_1$

$= ({}^6C_3 + 5)$ ways

⇒ RAVINDRA ⇒ choose two

1) 2 same – 2

2) 2 different – ${}^6C_2$

Total = $(2 + {}^6C_2)$

$\Rightarrow \boxed{{}^nP_r = {}^nC_r \times r!}$

2. RECURRANCE RELATIONS

1. INTRODUCTION

$T(n) = T(n-1) + T(n-2) + (T(n-3)) + K.$

$T(n) = aT(n/b) + K \rightarrow$ masters theorem

2. RECURRANCE RELATION AND SOLUTION

Sequence: Any order in which you write a number.

$T(n) = 0, 2, 4, 6, 8, 10 - - -$

$n = 0, 1, 2, 3, 4, 5$

$T(0)=0, \ T(1)=2, \ T(2)=4, \ T(3)=6.$

Recurrance Relation: It is a relation, in which we are possible to write present the nth term / Any term in the sequence as an mathematic Expression of previous terms then it is called RR.

1) $\boxed{T(n) = T(n-1)+2} \ (n \geq 2)$ [for above example)]

$\left.\begin{array}{l} T(0)=0. \\ T(1)=2 \end{array}\right\}$ — Initial conditions

The main aim of topic RR is to find such sol to the RR's.

sol to above RR $T(n) = 2n$.

LHS of RR $= T(n) = 2n$

RHS of RR $= T(n-1)+2 = 2(n-1)+2$

$= 2n.$

2) Seq: $1, 3, 5, 7, 9, 11 - - -$

$n = 0, 1, 2, 3, 4, 5$

$T(0)=1, \ T(1)=3, \ T(2)=5 \ \boxed{T(n-1)+2 = T(n)}$

$\boxed{T(n) = 2n+1} \ n \geq 1$

$T(0)=1$ (Initial condition)

$T(n) = 2n+1$

LHS of RR $= T(n) = 2n+1$

RHS of RR $= T(n-1)+2$

$= 2(n-1)+1+2$

$= 2n+1.$

Sequence: 1, 2, 4, 8, 16 - - - -

n: 0, 1, 2, 3, 4

$T(0)=1$, $T(1)=2$, $T(2)=4$, $T(3)=8$ - -

$\boxed{T(n) = 2T(n-1); n>0}$

$T(n)=0$, $n=0$. → [Initial condition]

Sol: $T(n) = 2^n$

LHS = $T(n) = 2^n$

RHS = $2T(n-1)$

$= 2^1 2^{n-1}$

$= 2^n$ // LHS = RHS

3. EXAMPLES

For the recurrance relation $T(n) = 2T(n-1) - T(n-2)$, $n \geq 2$, Find whether the following are solutions?

a) $T(n) = 3n$ b) $T(n) = 2^n$ c) $T(n) = 5$.

opvm,

(a) $T(n) = 3n$.

$T(n) = 2T(n-1) - T(n-2)$

LHS = $T(n) = 3n$

RHS = $2T(n-1) - T(n-2)$

$= 2[3(n-1)] - [3(n-2)]$

$= 6(n-1) - [3n-6]$

$= 6n - \cancel{6} - 3n + \cancel{6} = 3n$. ✓ This is soln

(or)

$T(n) = 3n$

$T(0) = 0$

$T(1) = 3$

$T(2) = 6$

$T(3) = 9$

$T(2) = 2T(1) - T(0)$

$= 2 \times 3 - 0$

$= 6$ ✓

$T(3) = 2[T(2) - T(1)]$

$= 2(6) - 3$

$= 12 - 3 = 9$.

b) $T(n) = 2^n$

$T(0) = 1$

$T(1) = 2$ //

$T(2) = 4$ ← Not equal → 3

$T(3) = 8$

$T(2) = 2T(1) - T(0)$

$= 2(2) - 1$

∴ 2^n is not solution.

(c) $T(n) = 5$

$T(0) = 5$

$T(1) = 5$

$T(2) = 5$

$T(3) = 5$

$T(2) = 2T(1) - T(0) = (2 \times 5) - 5 = 5$ //

$T(3) = T(3) = 2T(2) - T(1)$

$= 2 \times 5 - 5 = 5$ //

4. GATE 2004 QUESTION

The Recurrance relation $T(n) = 2T(n-1) + n,\ n \geq 2$ Evaluates to
$T(1) = 1$

a) $2^{n+1} - n - 2$ b) $2^n - n$ c) $2^{n+1} - 2n - 2$ d) $2^n + n$.

$T(1) = 1$

opA:- $2^{1+1} - 1 - 2 = 1$ c) $2^2 - 2 - 2 = 0$ X

opB: $2^1 - 1 = 1$ d) $2^n + n = 2^1 + 1 = 3$ X

$T(2) = 2T(1) + 2 \Rightarrow 2(1) + 2 = 4$

opA: $2^3 - 2 - 2 = 4$ ✓

opB: $2^2 - 2 = 2$

$\therefore$ $\boxed{\text{opA: } 2^{n+1} - n - 2}$

5. GATE 2002 QUESTION

The solution to the Recurrance Relation $T(2^k) = 3T(2^{k-1}) + 1$
$T(1) = 1$ is

a) 2^k b) ✓ $(3^{k+1} - 1)/2$ c) $3^{\log_2 k}$ d) $2^{\log_3 k}$

$T(1) = 1$

Q) Now if the question is $T(2^k) = 3T(2^{k-1}) + 1$ and options are given in Bigoh notations then how to solve them is.

$T(2^k) = 3T(2^{k-1}) + 1 \Rightarrow$ let $2^k = n$

$2^k/2 = n/2 \Rightarrow \boxed{2^{k-1} = n/2}$

$T(n) = 3T(n/2) + 1 \longrightarrow T(n) = aT(n/b) + \Theta(n^k \log^p n)$

$a = 3,\ b = 2,\ k = 0,\ p = 0.$

$a > b^k \Rightarrow T(n) = \Theta(n^{\log_b a})$

$\Rightarrow T(n) = \Theta(n^{\log_2 3})$

$= \Theta((2^k)^{\log_2 3}) = \Theta(2^{k \cdot \log_2 3}) = \Theta(2^{\log_2 3^k})$

$\boxed{T(n) = \Theta(3^k).}$

Given:- $T(2^k) = (3^{k+1} - 1)/2$

$T(1) = 1 \Rightarrow$ put $k = 0$ in $T = 1$

$= (3^1 - 1)/2 = 1$ ✓

~~LHS: $T(2^k) = (3^{k+1} - 1)/2$~~

~~RHS: $3T(2^{k-1}) + 1$~~

~~$= 3\left[\left(\frac{3^{k+1} - 1}{1}\right)/2\right]/2 = \frac{\frac{3}{2}\left(\frac{3^{k+1} - 1}{1}\right)}{2} = \frac{3}{4}($~~

Now, if the questions are given of this form $T(n) = 3T(\sqrt{n}) + 1$ then convert it as

$T(n) = 3T(\sqrt{n}) + 1$

let $n = 2^k \Rightarrow T(2^k) = 3T(2^{k/2}) + 1$

$\Rightarrow$ let $2^k = p$ then, $\boxed{T(p) = 3T(p/2) + 1}$

↳ Apply master theorem

6. GATE 2008 QUESTION

When $n = 2^{2k}$ for some $k \geq 0$, the reccurrance relation $T(n) = \sqrt{2}\,T(n/2) + \sqrt{n}$ evaluates to

$T(1) = 1$

a) $\sqrt{n}(\log n + 1)$ b) $\sqrt{n}\log n$ c) $\sqrt{n}\log\sqrt{n}$ d) $n\log\sqrt{n}$

$T(1) = 1 \Rightarrow T(2) = \sqrt{2}\,T(1) + \sqrt{2}$

$= 2\sqrt{2}$

opA: $\sqrt{n}(\log n + 1) = T(n)$

$T(1) = \sqrt{1}(\log 1 + 1) = 1$ ✓.

$T(2) = \sqrt{2}(\log 2 + 1)$

$= \sqrt{2}(2) = 2\sqrt{2}$ ✓

8. GATE 2008 QUESTION

Let x_n denote the no. of binary strings of length 'n' that contain no consecutive 0's. Which of the following reccurrance relation does x_n satisfy

a) $x_n = 2x_{n-1}$ ~~b)~~ $x_n = x_{n/2} + 1$ c) $x_n = x_{n/2} + n$ ~~d)~~ $x_n = x_{n-1} + x_{n-2}$

opvm

x_1 = The no. of Binary strings that contain no consecutive ones $= \{1, 0\}$

$\Rightarrow x_1 = 2$

$x_2 = \{01, 10, 11\}$

$= 3.$

$x_3 = \{5\}.$

opa: $x_1 = 2x_0 = 0$ ✗

opB: $x_2 = x_{2/2} + 1 = 2 + 1 = 3.$ ✗

opc: $x_2 = x_{2/2} + 1 = x_2 = 2 + 2 = 3 + 1 = 4$ ✗

opD: $x_2 = x_{one} + x_0 = 2 + 0 = 2.(\neq 3)$ ✗

op.D: $x_3 = x_2 + x_1$

$= 3 + 2$

$= 5$ ✓.

SETS

1. SETS AN

Set: well

Ex: A =

S =

Nullset: S

subset: I

of B.

A

Note: Fo

Proper subs

proper su

Not

2. POWER

Denoted by

⇒ If 'A' is

power s

Ex: A =

subs

⇒ If a se

9. GATE 2015 QUESTION

Let a_n Represent the no. of bit strings of length 'n' containing 2 consecutive 1's what is the recurrance relation for a_n?

a) $a_{n-2} + a_{n-1} + 2^{n-2}$ c) $a_{n-2} + 2a_{n-1} + 2^{n-2}$

b) $2a_{n-2} + a_{n-1} + 2^{n-2}$ d) $2a_{n-2} + 2a_{n-1} + 2^{n-2}$

$a_1 = 0$

$a_2 = 1$ {11}

$a_3 = 3 = \{011, 110, 111\}$

$a_4 = 8 = \{ - - \}$

opA: $a_3 = 8$ (from side analysis)

$a_n = a_{n-2} + a_{n-1} + 2^{n-2}$

$a_3 = a_1 + a_2 + 2 = 0 + 1 + 2 = 3$ ✓

opB: $a_1 + 2a_2 + 2 = 4$ ✗

opc: $2a_1 + a_2 + 2^1 = 3$ ✓

opd: $a_3 = 4$ ✗

Now, $a_4 = 8$ — opA: $a_2 + a_3 + 2^2 = 8$ ✓   ∴ $\boxed{\text{opA}}$

— opc: $2\times1 + 3 + 4 = 9$

3. GENERATING FUNCTIONS

1. INTRODUCTION

Let us say we have a series $a_1 a_2 a_3 a_4 a_5$ now what is the generating function for the sequence. GF is nothing but we take each ele in the sequence and assign some weights. Now, the GF for the above sequence will be $a_1 x^0 + a_2 x^1 + a_3 x^2 + a_4 x^3 + a_5 x^4$

Now, The seq: 1,1,1,1,1,1

GF: $1x^0 + 1x^1 + 1x^2 + 1x^3 + 1x^4 + x^5$

$= 1 + x + x^2 + x^3 + x^4 + x^5$.

Seq:- 1,1,1,1 - - -

GF $= 1\cdot x^0 + 1\cdot x^1 + 1\cdot x^2 + 1\cdot x^3 + - - -$

GF $= 1 + x + x^2 + x^3 + - - -$

2. EXAMPLE - 1

1> $1\cdot x^0 + 1\cdot x^1 = 1 + x = \frac{(x^2-1)}{(x-1)} = \left[\frac{1-x^2}{1-x}\right]$ Given seq = 1,1

2> Seq: 1,1,1

$1\cdot x^0 + 1\cdot x^1 + 1\cdot x^2 = 1 + x + x^2 = 1 + x + x^2 * \frac{(x-1)}{(x-1)} = \frac{(x^3-1)}{(x-1)} = \left[\frac{1-x^3}{1-x}\right]$

3> 1, 1, 1 - - - n 1's

GF: $1 + x + x^2 + \cdots + x^n = \frac{x^n - 1}{x - 1} = \left[\frac{1 - x^n}{1 - x}\right]$

4. INTRODUCTION TO PROPOSITIONAL CALCULUS. ②

1.1 INTRODUCTION

(29)

Proposition: Any declarative statement with a truth value assigned to it is called proposition. Ex: 1) My name is Ravindra Babu Ravula.

2) I am hungry [yes/no is truth value]

3) $x = y+1$ [No truth value]

2. CONNECTIVES $\wedge$ AND $\vee$ AND $\sim$

Propositional variable: Variable used to denote a proposition is called pv.

Connectives: These are the things which connect more than one propositional variable (propositions)

p: My name is RBR

Negation: $\sim p$: My name is not RBR

: It is not the case that my name is not RBR.

$\wedge$: p: I got 80% marks, Q: I got A Grade } $P \wedge Q$ = I got 80% marks AND I got 'A' grade.

$\vee$: P:, Q: } $P \vee Q$ = I got 80% marks OR A grade.

P.	$\sim$P.
0	1
1	0

P	Q	$P \wedge Q$.
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
F	T	T
T	F	T
F	F	F.

3. IMPLICATION

P	Q	$P \to Q$.
T	T	T
T	F	F
F	T	T
F	F	T

p: you get 60 marks in Gate

q: you will goto iit's.

$p \to q$: If you get 60 marks in Gate then you will goto IIT's.

P: Today is monday (T/F)

Q: $5 \times 2 = 10$ (T)

P	Q	
T	T	$\to$ T
F	T	$\to$ T

$\therefore$ $p \to Q$ is always a

The various ways of Representing $p \to Q$ are:

1> If p then Q. → A necessary condition for 'p' is Q.

2> 'p' is sufficient for Q. → Q unless $\sim p$

3> Q if 'p'. → 'p' implies Q.

4> Q when 'p'. → 'p' only if Q.

→ A sufficient condition for 'Q' is 'p'.

→ 'Q' whenever 'p'.

→ 'Q' is necessary for 'p'.

→ 'Q' follows from 'p'.

4. QUESTION ON IMPLICATION

$p \to Q$: If today is friday then $2\times3=6$.

Converse: $Q \to p$: If $2\times3=6$ then today is friday

Contrapositive: $\sim Q \to \sim p$: If $2\times3\neq6$ then today is not friday

Inverse: $\sim p \to \sim Q$: If today is not friday then $2\times3\neq6$.

i> $p \to Q \neq \;\to p$.

$p \to Q \cong [\sim p \vee Q]$

$\cong (\sim Q \to \sim p)$

$p \to Q \cong \sim Q \to \sim p$ (conditional & contrapositive)

$q \to p \cong \sim p \to \sim Q$ (Inverse and Converse)

5. BI- CONDITIONAL

P	Q	$P \Leftrightarrow Q$	$(p\to Q)\wedge(Q\to p)$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	T	T

$\therefore p \Leftrightarrow Q \cong (p \to Q) \wedge (Q \to p)$

Various Representations of Biconditional are: if and only iff

iff

it is necessary and sufficient.

precedence of operations : 1> Negation

2> Conjunction

3> Disjunction

4> Implication

5> Bi Implication

$\Rightarrow \sim p \wedge q \vee i$

$\Rightarrow ((\sim p) \wedge q) \vee i$

6. EXAMPLE 1

You can acess the internet only if you are a computer science major or you are not a freshman.

(31)

There are 3 statements here

p: you can acess the internet $\rightarrow p$.
q: you are a computer science major
R: you are not a freshman.

$\left.\begin{matrix} q \\ r \end{matrix}\right\} (q \vee r)$

$\boxed{p \rightarrow (q \vee r)}$

$\Rightarrow$ what ever before "only if" is called Hypothesis and what is given after that is called conclusion.

7. EXAMPLE 2

$\underbrace{\text{You cannot ride the roller coaster}}_{\sim p}$ if $\underbrace{\text{you are under 4 feet tall}}_{Q}$ unless $\underbrace{\text{you are older than 16 years.}}_{r}$

p: you can ride the roller Coaster
q: you are under 4 feet tall
r: you are older than 16 years.

Now, we can replace 'unless' with 'And' and Negate the statement 'r'.

$\therefore (Q \wedge \sim r)$

$p \rightarrow Q$ (r if p)
r → Conclusion, p → Hypothesis (After if)

$(\sim p)$ → Before if = conclusion

$(Q \wedge \sim r)$ → present after if → Hypothesis.

$\therefore$ Final result = Hypothesis → Conclusion

$\Rightarrow \boxed{(Q \wedge \sim r) \rightarrow p.}$

8. EXAMPLE - 3

$\underbrace{\text{The automated reply cannot be sent}}_{\sim P}$ when $\underbrace{\text{the file system is full}}_{Q}$

$\sim p$ when q.

p: The automated reply canbesent

Q: The file system is full

$\therefore \boxed{q \rightarrow \sim p}$

if $p \rightarrow Q$. = Q when P

Q: Conclusion, P: Hypothesis

9. CONSISTENT SYSTEM

Determine whether these specifications are consistent.

→ The diagnostic message is stored in the buffer or it is retransmitted

→ The diagnostic message is not stored in the buffer

→ "If the diagnostic message is stored in the buffer, then it is retransmitted.

1> $\underbrace{\text{The diagnostic message is stored in the buffer}}_{P}$ or $\underbrace{\text{it is retransmitted}}_{Q}$ $= p \vee Q$.

2> The diagnostic message is not stored inbuffer $(\neg p)$

3> If the diagnostic message is stored inthe buffer, then it is retransmitted. $(p \rightarrow Q)$

$\therefore$ 1> $(p \vee Q)$
2> $(\sim p)$
3> $(p \rightarrow Q)$

Now we should find some assignment of truth values to 'p' and 'Q' such that all the '3' will be True simultaneously

$p \vee Q$. $\quad \sim p \quad$ $p \rightarrow Q$.

$\sim p$ ↓ This should be true ⇒ $\sim p = T$

⇒ $\boxed{p = F}$

Now, $p \vee Q$ should be True ⇒ $\boxed{p = F}$ already known

$\boxed{q = T}$

10. EQUIVALENCES

1> $p \to Q = \sim p \vee Q$

2> $p \to Q = \sim q \to \sim p$

3> $p \vee Q = \sim p \to Q$

4> $p \wedge Q = \sim (Q \to \sim p)$

5> $\sim (p \to Q) = p \wedge \sim q$

6> $(p \to Q) \wedge (p \to r) = p \to (Q \wedge r)$

7> $(p \to r) \wedge (q \to r) = (p \vee Q) \to r$

8> $(p \to Q) \vee (p \to r) = p \to (q \vee r)$

9> $(p \to r) \vee (q \to r) = (p \wedge Q) \to r$

10> $(p \leftrightarrow q) = (p \to q) \wedge (q \to p)$

11> $\sim (p \leftrightarrow q) \equiv (p \leftrightarrow \sim q)$

11. DE MORGANS LAW

1> $\sim (p \vee Q) \equiv \sim p \wedge \sim Q$.

2> $\sim (p \wedge Q) = \sim p \vee \sim Q$.

Ex: Ravi has a computer and a phone

$\Downarrow$

If i apply Negation to this what will be the output??

let p: Ravi has a computer
Q: Ravi has a phone

Now, $\sim (p \wedge Q) = \sim p \vee \sim Q$.

$\to$ Ravi doesnot have a ~~phone~~ computer or Ravi doesnot have a phone

12. PROPOSITIONAL FUNCTION

$\to$ proposition is a statement to which we can assign a truth value.

Ex: 2 is greater than 10 (False) $\leftarrow$ proposition.

$\{\underset{\text{variable}}{x}\ \underbrace{\text{is greater than 10}}_{\text{predicate}}\}$ – cannot take T/F unless the value of x is known.

$\downarrow$

This is called propositional Function. $p(x)$

$\therefore$ $p(x)$: x is greater than 10
$p(2)$: 2 is >10 (F)
$p(12)$: 12 is >10 (T)

$\to$ The propositional function transforms itself into proposition.

$p(x)$ / $p(x,y)$ computer x is connected to network y.

13 QUANTIFIERS

$p(x): x > 2$

Domain: $\{1,2,3,4\}$ $p(1)=F$, $p(2)=F$ $p(3)=T$, $p(4)=T$

Now, $\forall x\, p(x)$ is true, if $p(x)$ is true for all values of x.

$\therefore$ $\boxed{\forall x p(x) = \text{False}}$ in this case

$\forall x$ → Universal Quantifier / universal Quantification.

Now, let $p(x) = x > 2$

Domain $= \{1,2,3,4\}$

$p(1)=F$ $p(2)=F$ $p(3)=T$, $p(4)=T$ $\therefore$ $\exists x\, p(x)$ is true if there exists atleast one value of x in the domain for which $p(x) = \text{True}$

In this case $\boxed{\exists x p(x) = \text{True}}$ → Existential Quantifier.

14. RELATION BETWEEN THE TWO QUANTIFIERS

If $\forall x p(x)$ is true then $\exists x\, p(x)$ is True

If $\forall x p(x)$ is false then $\exists x\, p(x)$ is (false/True)

If $\exists x p(x)$ is true then $\forall x p(x)$ is true

If $\exists x p(x)$ is false then $\forall x p(x)$ is false

$p(x) = x > 3$

Domain $= \{2,3,4\}$

$\exists x p(x) = T$

$\forall x p(x) = F$

→ The precedence of $\forall, \exists$ will be higher than any other logical connectives.

$$\forall x p(x) \vee Q(x) \Rightarrow [\forall x p(x)] \vee Q(x).$$

15. DISTRIBUTING QUANTIFIERS

1> $\forall x(x+y+z=10) \cong (\forall x(x)) + y + z = 10$

2> $\exists x p(x) \vee \forall x Q(x) \cong [\exists x p(x)] \vee [\forall x Q(x)]$

3> $\forall x (p(x) \wedge Q(x))$

Domain $= \{x_1 x_2\}$ $=$ $p(x_1) \wedge Q(x_1) \wedge Q(x_1) \wedge Q(x_2)$ $\therefore \forall$ can be

④ $\forall x$ cannot be "distributed over disjunction."

$$\left.\begin{array}{l}\forall x\,(p(x) \vee Q(x)) = [p(x_1) \vee Q(x_1)] \wedge [p(x_2) \vee Q(x_2)] \\ D: \{x_1, x_2\} \\ [\forall x\, p(x)] \vee [\forall x\, Q(x)] = [p(x_1) \wedge p(x_2)] \vee [Q(x_1) \wedge Q(x_2)]\end{array}\right\}\text{Not Equiv.}$$

⑤ $\exists x(p(x) \vee Q(x)) = [\exists x\, p(x)] \vee [\exists x\, Q(x)] \rightarrow$ There exists can be distributive over disjunction.

16. QUANTIFIERS WITH NEGATION

1) Every student in the class has taken a course in CS

$p(x)$: x has taken a course in CS

$\forall x p(x)$:

x = students of the class.

Now, $\sim$(statement 1) = There exist atleast one student in the class who has not taken the course in CS.

$$\boxed{\sim(\forall x\, p(x)) \cong \exists x(\sim p(x))} \qquad \boxed{\sim \exists x\, p(x) \cong \forall x \sim p(x)}$$

17. EXAMPLES ON NEGATING THE QUANTIFIERS

→ There is an honest politician

$H(x)$: x is honest

$\exists x H(x)$: There is an honest politician

x: {All politicians}

Now, $\sim(\exists x\, H(x)) = \forall x \sim(H(x))$

= All the politicians are dishonest.

→ All Americans eat cheese burgers.

$C(x)$: x eats cheese burgers $\Rightarrow \forall x C(x)$ and x: {All Americans}, Now

$\sim(\forall x\, C(x)) = \exists x \sim(C(x)) \Rightarrow$ "There exists an American who dont eat cheese burgers."

18. IMPORTANT EXAMPLES

1> $\sim \forall x(p(x) \wedge Q(x))$

$\Rightarrow \sim \forall(x) [R(x)]$ where $R(x) = p(x) \wedge Q(x)$

$\Rightarrow \exists x \sim (R(x))$

$\Rightarrow \exists x \sim (p(x) \wedge Q(x))$

$\Rightarrow \exists x [\sim p(x) \vee \sim Q(x)]$

2> $\sim \forall x(p(x) \rightarrow Q(x))$

$\Rightarrow \sim \forall x(\sim p(x) \vee Q(x))$

$\Rightarrow \exists x [p(x) \wedge \sim Q(x)]$ Statement (4)

OPA: 3> $\sim \exists x(p(x) \wedge Q(x)) = \forall x(\sim p(x) \vee \sim Q(x)) \equiv \forall x(p(x) \rightarrow \sim Q(x))$

4> $\forall x(p(x) \rightarrow \sim Q(x)) = \forall x(\sim p(x) \vee \sim Q(x))$

19. TRANSLATING ENGLISH STATEMENTS TO PROPOSITIONAL FUNCTIONS

Every student in the class has taken the course operating system.

$O(x)$: x has taken the course OS.

x : {students in the class}

$\forall x\, O(x)$

Now let $S(x)$: x is a student in the class.

$\therefore$ If 'x' is a student in the class then he has taken OS

$\forall x(S(x) \rightarrow O(x))$ (domain is expanded Here)

$\forall x$ — For all people in the world

$S(x) \rightarrow O(x)$ — If he has a student in the class then he has taken the course OS.

x = {All the people in the world)

Hint: Now, if you see the word Every in the Question the option Ans must contain $\forall$ and $\rightarrow$.

Now, $\sim \forall x(S(x) \rightarrow O(x)) = \exists x(S(x) \wedge \sim O(x)) \rightarrow$ Check

20. TRANSLATION CONTINUED

Some student in the class has taken OS course

$O(x)$: x has taken CS.

$\Rightarrow \exists x\, O(x)$

x: students in the class

Now, $S(x)$: x is student in the class

$\exists x\,(S(x) \wedge O(x))$ $\Rightarrow$ Now, if you see "some" then Ans should contain ($\exists$ and AND).

Now, $\sim [\exists x (S(x) \wedge O(x)]$

$\sim$(some student in the class has taken OS course) = (No student in the class has taken OS course)

$\therefore \sim [\exists x\, (S(x) \wedge O(x)]$

$= \forall x\, [\sim S(x) \vee \sim O(x)]$

$= \forall x\, [S(x) \rightarrow \sim O(x)]$